ASSIGNMENT 1
UNIFORM DISTRIBUTION THEORY 2021
DUE DATE: MARCH 17, 2021

Exercise 1. Show that the sequence
$$\{\frac{0}{1}, \frac{0}{2}, \frac{1}{2}, \frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{0}{5}, \dots\}$$
is uniformly distributed modulo one.
Hint: Use Weyl's criterion.

Exercise 2. Let $\varphi = (1+\sqrt{5})/2$ be the golden mean. Show that the sequence $\{\varphi^n : n = 1, 2, 3, \dots\}$ is <u>not</u> dense modulo one, in fact that the only limit points are 0 and 1.
Hint: Let $\widetilde{\varphi} = (1-\sqrt{5})/2$. Show $U_n := \varphi^n + (\tilde{\varphi})^n$ is an integer, and hence $\mathrm{dist}(\varphi^n, \mathbb{Z}) \to 0$ as $n \to \infty$.

Exercise 3. Show that as $N \to \infty$,
$$\sum_{n=1}^{N} \frac{\log n}{n} = \frac{1}{2}(\log N)^2 + O(1).$$
Hint: Use Euler's summation formula.